MTTC Secondary Math Review Solutions

1. The correct answer is D. The principle of mathematical induction requires one to first complete a basis step, proving the statement is true for n=1 or some other value of n, and then to complete the inductive step, where the statement is assumed to be true for $n = k$ and then shown to be true for $n = k$+1.

2. The correct answer is A. The pizza is taken out of the freezer so its initial temperature will be 0˚C or colder. While it is in the oven its temperature will rise. When it is taken out of the oven it will cool down until it reaches room temperature. B is wrong because it shows a constant rate of heating and cooling, which is not true. The rate will depend on the temperature of the pizza. C is showing time as a function of temperature. D is wrong because it shows the pizza cooling all the way back to freezing.

3. The correct answer is B. $g(-2) = 2(-2) + 3 = -1$. And

$$(f \circ g)(-2) = f\big(g(-2)\big) = f(-1) = 3(-1)^2 - 4(-1) = 3 + 4 = 7$$

4. The correct answer is B. First take the derivative using the product rule, which is $f'(x) = \sin x + x\cos x$. Then set the derivative equal to 0 and solve for x to obtain $\sin x + x\cos x = 0 \Leftrightarrow x\cos x = -\sin x \Leftrightarrow x = \frac{-\sin x}{\cos x} = -\tan x$.

5. The correct answer is C. Matrix multiplication of a 3x3 matrix multiplied by a 3x1 matrix produces a 3x1 matrix. To do the multiplication multiply each entry of row one by its corresponding entry of column one, in this case x, and add them up to get row one of the resulting matrix, b. Repeat for the other the other two rows. So the product of $\begin{pmatrix} 1 & 3 & 1 \\ 2 & 1 & 4 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ produces $\begin{pmatrix} (1\cdot 1) + (3\cdot 2) + (1\cdot 3) \\ (2\cdot 1) + (1\cdot 2) + (4\cdot 3) \\ (1\cdot 1) + (1\cdot 2) + (1\cdot 3) \end{pmatrix} = \begin{pmatrix} 10 \\ 16 \\ 6 \end{pmatrix}$

6. The correct answer is D. The volume of a cube is $V = x^3$. Relative percent error is the change in volume with respect to change in length divided by the volume, or $\frac{\Delta V}{V} \cdot 100\% \approx \frac{dV}{V} \cdot 100\% = \frac{3x^2dx}{x^3} \cdot 100\% = \frac{3(5)^2(.1)}{5^3} \cdot 100\% = \frac{3(.1)}{5} \cdot 100\% = 6\%$

7. The correct answer is D. If ∞ is substituted for each x the result is $\frac{\infty}{\infty}$, an indeterminate form. This indeterminate limit can be solved by dividing the numerator and denominator by x^3 or by using L'Hospitals rule. Dividing by x^3 produces $\lim_{x\to\infty}\frac{3x^2/x^3+5x/x^3}{x^3/x^3+4x^2/x^3}=\lim_{x\to\infty}\frac{3/x+5/x^2}{1+4/x}=\frac{0}{1}=0$. Using L'Hospital's rule involves taking the derivative of both the numerator and denominator separately to get $\lim_{x\to\infty}\frac{6x+5}{3x^2+8x}$. Upon substitution of ∞ for x, the result is once again the same indeterminate form $\frac{\infty}{\infty}$. So, L'Hospital's rule must be used once more to get $\lim_{x\to\infty}\frac{6}{6x+8}$. This time upon substitution of ∞ for x the result is $\frac{6}{\infty}=0$.

8. The correct answer is B. The slope of the tangent line is the value of the derivative of $f(x)$ at $x=3$. $f'(x)=6x+4$ so $f'(3)=6(3)+4=18+4=22$. Therefore, $y=22x+b$. So B is the only possible choice. However, we can check that it is correct as follows. The value of $f(x)=3x^2+4x+2$ when $x=3$ is $f(3)=3(3)^2+4(3)+2=27+12+2=41$. Thus, the point on the graph that the tangent line goes though is $(3,41)$. To determine b substitute the point $(3,41)$ in $y=22x+b$. $41=22(3)+b\leftrightarrow 41=66+b\leftrightarrow -25=b$. The equation of the tangent line at $x=3$ is $y=22x-25$.

9. The correct answer is B. Since this quadratic is not factorable, a method to solve it is to use the quadratic formula, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$. Upon substitution, $x=\frac{-2\pm\sqrt{2^2-4(1)(2)}}{2(1)}=\frac{-2\pm\sqrt{4-8}}{2}=\frac{-2\pm\sqrt{4}i}{2}=\frac{-2\pm 2i}{2}=-1\pm i$.

10. The correct answer is D. This problem can be solved by elimination, substitution, or by matrix row reduction. To solve the system using row reduction, first put the equations

$$\begin{aligned}3x+2y+z&=2\\ 2x-y-z&=1\\ x+y&=1\end{aligned}$$

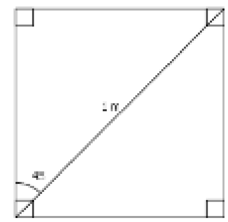
into matrix form and reducing the matrix to row reduced echelon form as follows

$$\begin{bmatrix}3&2&1&2\\2&-1&-1&1\\1&1&0&1\end{bmatrix}\overrightarrow{R_1\leftrightarrow R_3}\begin{bmatrix}1&1&0&1\\2&-1&-1&1\\3&2&1&2\end{bmatrix}\overrightarrow{\begin{matrix}-2R_1+R_2\to R_2\\-3R_1+R_3\to R_3\end{matrix}}\begin{bmatrix}1&1&0&1\\0&-3&-1&-1\\0&-1&1&-1\end{bmatrix}\overrightarrow{R_2\leftrightarrow R_3}$$

$$\begin{bmatrix}1&1&0&1\\0&-1&1&-1\\0&-3&-1&-1\end{bmatrix}\overrightarrow{-R_2\to R_2}\begin{bmatrix}1&1&0&1\\0&1&-1&1\\0&-3&-1&-1\end{bmatrix}\overrightarrow{\begin{matrix}-R_2+R_1\to R_1\\3R_2+R_3\to R_3\end{matrix}}\begin{bmatrix}1&0&1&0\\0&1&-1&1\\0&0&-4&2\end{bmatrix}\overrightarrow{\frac{-1}{4}R_3\to R_3}\begin{bmatrix}1&0&1&0\\0&1&-1&1\\0&0&1&\frac{-1}{2}\end{bmatrix}$$

$$-R_3 + R_1 \to R_1 \quad R_3 + R_2 \to R_2 \to \begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{-1}{2} \end{bmatrix}$$. Thus $x = \frac{1}{2}, y = \frac{1}{2}, z = \frac{-1}{2}$

11. The correct answer is D. Since the square is inscribed in a circle whose diameter is 1 m, the diagonal of the square will be 1 m. To find the length of the side of the square, we use trigonometry. $\sin 45° = \frac{x}{1} \leftrightarrow \frac{\sqrt{2}}{2} = x$. Therefore, the area will be $x^2 = \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{2}{4} = \frac{1}{2}$



12. The correct answer is B

$$f(1) = 3f(0) - 2 = 3(3) - 2 = 9 - 2 = 7$$
$$f(2) = 3f(1) - 2 = 3(7) - 2 = 21 - 2 = 19$$
$$f(3) = 3f(2) - 2 = 3(19) - 2 = 57 - 2 = 55$$
$$f(4) = 3f(3) - 2 = 3(55) - 2 = 165 - 2 = 163$$

13. The correct answer is C.
$f(x) = \frac{4x-1}{3-2x} \leftrightarrow y = \frac{4x-1}{3-2x} \leftrightarrow y(3-2x) = 4x - 1 \leftrightarrow 3y - 2xy = 4x - 1 \leftrightarrow -2xy - 4x = -3y - 1 \leftrightarrow 2xy + 4x = 3y + 1 \leftrightarrow x(2y+4) = 3y + 1 \leftrightarrow x = \frac{3y+1}{2y+4}$. Now, we must exchange the values of x and y to obtain $y = \frac{3x+1}{2x+4} \leftrightarrow f^{-1}(x) = \frac{3x+1}{2x+4}$

14. The correct answer is B. Assume 1 is the total distance across the pond. Let d_n = the remaining distance after the n^{th} jump. So $d_1 = \frac{3}{8}$, $d_2 = \frac{3}{8} d_1 = \frac{3}{8} \cdot \frac{3}{8} = \left(\frac{3}{8}\right)^2$, $d_3 = \frac{3}{8} d_2 = \frac{3}{8}\left(\frac{3}{8}\right)^2 = \left(\frac{3}{8}\right)^3$, …
So the total distance covered is $\frac{5}{8} + \frac{5}{8} d_1 + \frac{5}{8} d_2 + \frac{5}{8} d_3 \ldots = \frac{5}{8} + \frac{5}{8} \cdot \frac{3}{8} + \frac{5}{8} \cdot \left(\frac{3}{8}\right)^2 + \frac{5}{8} \cdot \left(\frac{3}{8}\right)^3 \ldots = \frac{5}{8}\left[1 + \frac{3}{8} + \left(\frac{3}{8}\right)^2 + \left(\frac{3}{8}\right)^3 \ldots\right]$

15. The correct answer is A. The formula for the surface area and volume of a sphere are $A = 4\pi r^2$ and $V = \frac{4}{3}\pi r^3$ respectively. If the radius is 3, $A = 4\pi(3)^2 = 36\pi$ and $V = \frac{4}{3}\pi(3)^3 = 36\pi$. Therefore, the ratio is 1:1.

16. The correct answer is C. The value of $f(x) = \cos^{-1}(2 - \sqrt{2}\sin(x))$ when $x = \frac{\pi}{4}$ is $f\left(\frac{\pi}{4}\right) = \cos^{-1}(2 - \sqrt{2}\sin\frac{\pi}{4}) = \cos^{-1}\left(2 - \sqrt{2} \cdot \frac{\sqrt{2}}{2}\right) = \cos^{-1}(1) = 0.$

17. The correct answer is C. Each term in the sequence is the sum of the two previous terms. If the pattern continues until there are 10 terms, the sequence will be, 2,3,5,8,13,21,34,55,89,144
The highlighted terms are prime numbers. (Each term is divisible by only 1 and itself) There are

5 prime numbers in the first 10 terms.

18. The correct answer is C. The pool was filled at a steady rate at the beginning. Then, the volume starts to go down, so it must have been losing water. The volume then stayed constant for a period of time, so filling and emptying must have ceased. Lastly, the pool's volume increases at a steady rate.

19. The correct answer is C. Mass = Density ·Volume where density is given and volume can be found by using: $V = \frac{1}{3} \cdot \pi \cdot r^2 \cdot h$. Without a fundamental understanding of the sciences, a "Factor/Label" approach will also suffice (labels that appear in both the numerator and denominator cancel as do factors).

$$\text{Mass} = \left(\frac{2.6\text{g}}{\text{cm}^3}\right)\left(\frac{\pi\,(2.5\text{cm})^2 \cdot 8\text{cm}}{3}\right) = \left(\frac{2.6\text{g}}{\text{cm}^3}\right)\left(\frac{50\pi\ \text{cm}^3}{3}\right) = \frac{130\pi}{3}\text{grams}.$$

20. The correct answer is A. The height of the shaded area will be $f(x) - g(x)$. Therefore, $h(x) = 2x + 1 - (x^2 - 2x + 2) = 2x + 1 - x^2 + 2x - 2 = -x^2 + 4x - 1$.

21. The correct answer is C. The statement $P \wedge (\sim P)$ is always false, as may be quickly seen by constructing a truth table.

P	$\sim P$	$P \wedge (\sim P)$
T	F	F
F	T	F

A statement that is always false is a contradiction.

22. The correct answer is D. Note that all the y-values are 1, so first construct a function $f(x) = x(x + 2)(x - 3)(x - 5)$ that has zeros of –2, 0, 3, 5. Then the function $f(x) + 1 = x(x + 2)(x - 3)(x - 5) + 1$ (shifted up one) unit will satisfy the conditions.

23. The correct answer is C, since it is the contrapositive of the statement (negated converse). If a statement is true, then the contrapositive of the statement is also true.

24. The correct answer is B. Recall that when finding roots of a polynomial with integer coefficients, all the possible rational roots have the form $\frac{c}{d}$, where c is a factor of the constant term, and d is a factor of the leading coefficient. Since 3 is not a factor of the constant 100, $\frac{3}{4}$ is not a possible root.

25. The correct answer is C. We wish to represent the solution set $-3 \leq x \leq 1$ as an absolute value inequality. Adding 1 to this equality gives$-2 \leq x + 1 \leq 2$, or equivalently $|x + 1| \leq 2$.

An alternative to a solution is note that $|x| \leq 2$ is an interval of length 4 centered at 0, but in this problem the interval is a length of 4 and centered at -1. Making use of the function shifting properties yields $|x+1| \leq 2$.

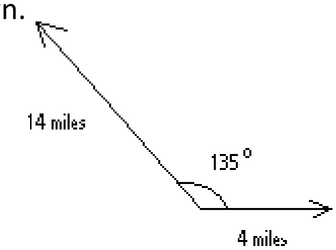
26. The correct answer is C. After the dry sponge has been dunked in water and taken out, each side length is scaled by a factor of $\frac{3}{2}$ from the original. Since there are 3 dimensions to the sponge, and each increases by a factor of $\frac{3}{2}$, the volume of the sponge is changed by a factor of $\left(\frac{3}{2}\right)^3 = \frac{27}{8}$.

27. The correct answer is D. The satellite will make a circular orbit of radius $3959\text{mi} + 200\text{mi} = 4159\text{mi}$. The time required to traverse the circle is $T = \frac{\text{distance}}{\text{rate}} = \frac{2\pi(4159)}{1090} = 23.97$.

28. The correct answer is D. The equation is that of a function, so it can't have x-axis symmetry as a possibility. Since, $f(-x) = f(x)$ is not true, it doesn't have y-axis symmetry. $f(-x) = -f(x)$ is not true, so it doesn't have origin symmetry.

29. The correct answer is C. The component of the 40 N force that is perpendicular to the boat is $40 \sin 45° = 20\sqrt{2}$ N. In order for the boat to not veer into the riverbank, the perpendicular component of the 50 N force must also be $20\sqrt{2}$ N. This means that $20\sqrt{2}\text{ N} = 50 \sin\theta$. Solving for θ yields 34.4°

30. The correct answer is C. After 2 hours the walker and biker have traveled 4 mi and 14 mi respectively. A basic illustration of the problem (not to scale) is shown.
The Law of Cosines is needed to complete the problem.
$a^2 = b^2 + c^2 - 2bc\cos\theta$



31. The correct answer is C. As the paper is folded, the length is repeatedly cut in half, thus decreasing exponentially, and the thickness is repeatedly multiplied by two, thus increasing the thickness exponentially.

32. The correct answer is B. The rate of change of the sphere's surface area may be expressed as:
$$\frac{dA}{dt} = \frac{d}{dt}(4\pi r^2) = 8ðr\frac{dr}{dt}$$
And the rate of change of the sphere's volume may be expressed as:
$$\frac{dV}{dt} = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = 4\pi r^2\frac{dr}{dt}$$
Combining these equations yields a relation between the volume rate of change and the area rate of change:
$$\frac{dV}{dt} = \frac{1}{2}r\frac{dA}{dt}$$
Plugging in $r = 4$ cm and $\frac{dA}{dt} = 2\ {\text{cm}^2}/{\text{sec}}$ gives $4\ {\text{cm}^3}/{\text{sec}}$.

33. The correct answer is D. First we use $y(t)$ to find the time that the ball is at its highest position. The time that maximizes $y(t)$ is $t = \frac{-10}{2(-5)} = 1 \text{ sec}$
Plugging this time back into $x(t)$ and $y(t)$ gives 8 ft and 105 ft respectively.

34. The correct answer is B. Remember that $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$. The side opposite of θ is x and the hypotenuse is z.

35. The correct answer is B. Remember that $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$. The side opposite of θ is x and the adjacent side is y.

36. The correct answer is C. Remember that $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$. The adjacent side of θ is y and the hypotenuse is z. Thus $\cos\theta = \frac{y}{z} = \frac{24}{25} = 0.96$

37. The answer is B. The side adjacent to the 30° angle is known and the opposite side is needed, Therefore $tan30° = \frac{\text{height}}{150}$. Therefore $\text{height} = 150\tan 30° = 86.60 \text{ ft.}$

38. The answer is D. To change degrees to radians, multiply the degree measure by $\frac{\pi}{180}$. $140 * \frac{\pi}{180} = \frac{140\pi}{180} = \frac{7\pi}{9}$

39. The answer is A. The pattern is a geometric sequence, where the nth term is $a_n = a_1 r^{n-1}$. Since the number of apples doubles for each iteration, $r = 2$. Since the first student brings one apple, $a_1 = 1$. So, $a_n = 1 \cdot 2^{11-1} = 2^{10} = 1024$.

40. The answer is C. This is a combination problem ${}_nC_r = \binom{n}{k} = \frac{n!}{r!(n-r)!}$. The number of combinations for the girls is $\binom{23}{9} = \frac{23!}{9!14!} = 817190$. The number of combinations for boys is $\binom{30}{9} = \frac{30!}{9!21!} = 14307150$. Since the choosing of the boys team and the choosing of the girls team are mutually exclusive (no overlap), the combinations are simply multiplied together, or $817190 \cdot 14307150$ =116,916,599,085

41. The answer is B. This is a geometric probability problem. $p = \frac{\text{area of circle}}{\text{area of square}} = \frac{\pi r^2}{(2r)^2} = \frac{\pi}{4} \approx 0.785$.

42. The answer is D. This is an example of the Central Limit Theorem. As the sample size increases the breadth of the distribution narrows. The sample standard deviation is $\frac{\sigma}{\sqrt{n}} = \frac{15}{\sqrt{5}}$.

43. The answer is D. This is a proportion problem. $\frac{500 \text{ miles}}{1 \text{ inch}} = \frac{x \text{ miles}}{24 \text{ inches}} \leftrightarrow x = 12{,}000 \text{ miles}$

44. The answer is C. The correlation coefficient is always between –1 and 1. It also has the same sign as the slope of the trend line. Since the trend has a negative slope, $-1 < r < 0$.

45. The answer is A. Since the confidence interval widens as the standard deviation increases and since B has a larger standard deviation, the confidence interval of B would be wider at identical

samples sizes and confidences levels. Increasing the number of items in the sample for B, or decreasing the confidence level, or both could narrow the interval of B to match that for A.

46. The correct answer is D. The other options include bias in selection of the participants of the survey. Only option D selects the survey participants randomly, ensuring that all students have an equal likelihood of being selected.

47. The answer is D. The box depicts the interquartile range, extending from the 1st quartile (25th percentile) to the 3rd quartile (75th percentile). The bar in the box is at the median. The lengths of the whiskers of a modified box plot are 1.5 times the interquartile range from the median. In a "normal" boxplot, the whiskers extend from the median to the minimum and maximum.

48. The correct answer is C. A tree diagram can show that at level one, four people are called. At level two, 16 people are called. At level three, 64 people are called. The number of people called increases by a factor of four at each level. The total number called is the summation of those called at that level and all previous levels.

49. The answer is C. A median bisects the side opposite the vertex. So, AD starts from vertex A and ends at point D which is the midpoint of BC.

50. The answer is D. In order for two triangles to be congruent, all corresponding parts must be congruent. Since both triangles are similar, all corresponding angles are the same measure. To use, ASA we need one pair of corresponding sides to be congruent, or equal in length. The important thing is to be certain that the pairs are corresponding.

51. The answer is A. When using a proof by contradiction, we assume the hypothesis, "the puppy is black", and use the negation of the conclusion as an assumption.

52. The answer is B. This is a conversion problem. In order to find how many bracelets Anaya can make, the 3 meters must be converted into inches by using the conversion ratio between centimeters and inches. $\frac{3\text{ meters}}{1} * \frac{100\text{ centimeters}}{1\text{ meter}} * \frac{1\text{ inch}}{2.54\text{ centimeters}} * \frac{1\text{ necklace}}{4\text{ inches}} = \frac{300}{10.16} \approx 29.52$ Since Anaya's bracelets must be 4 inches long, we must take the greatest whole number less than the actual answer, which is 29.

53. The answer is D. Let the base and height of triangle ΔABC be b and h. Since the triangles are similar and the scale factor is 2, the base and altitude of ΔDEF are $2b$ and $2h$. Upon substitution into the equation for the area of a triangle, the result is $A = \frac{2b(2h)}{2} = 4\left(\frac{bh}{2}\right)$.

54. The answer is C. The Pythagorean Theorem is used to prove right triangles by the lengths of the sides. Since only the vertices are given, the lengths, or distances, between the vertices need to be determined first before applying the Pythagorean Theorem.

55. The answer is B. In order to convert to alternate units, the conversion factor must cancel the units no longer needed. The correct set-up is as follows:

$$\frac{50\text{ kilograms}}{1} * \frac{2.2\text{ pounds}}{1\text{ kilogram}} * \frac{16\text{ ounces}}{1\text{ pound}}$$

Dividing by 16 would force the ounces to be on the bottom have pound squared on the top.

56. The correct answer is A. You treat ADE as one letter. Thus, you want to find the number of permutations of 6 objects. $P(6,6) = 6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$.

57. The correct answer is B. If 6 members must be chosen with an equal number of men and women, then 3 of the 12 females and 3 of the 14 males must be selected. The number of ways to pick 3 of the 12 females is $C(12,3) = \frac{12!}{3!(12-3)!} = 220$. The number of ways to pick 3 of the 14 male members is $C(14,3) = \frac{14!}{3!(14-3)!} = 364$. Using the product rule the total number of ways to pick the committee is $220 \cdot 364 = 80{,}080$.

58. The correct answer is D, all of the above. The set $\{-1,0,1\}$ is the set of all the integers that satisfy the descriptions in a, b, and c.

59. The correct answer is B. Since all the vertices have an even degree, we know an Euler circuit must exist, which is a path that starts and ends at the same vertex, using each side only once. Choices c and d do not use all of the sides and do not start and end at the same vertex so they are not an Euler Circuit. Choice b is one Euler circuit of the undirected graph.

60. The correct answer is D. A tree is a graph that is connected and contains no circuits. A spanning tree is a subset of edges that create a tree and use all the vertices of a graph. A minimum spanning tree is one of lowest weight or cost in a graph that has weighted edges. A minimum cost spanning tree can be found by selecting the edges of lowest weight in order, as long as a circuit is not created, until all the vertices are used and the subgraph is connected. A minimum spanning tree for this graph can be obtained by selecting edges *AE*, *DE*, *CB*, and *CE* (or *AB*), for a weight of 7.

61. The correct answer is A. S is the set of integers and R is the set of rational numbers, thus their intersection, which is defined as $\{x | x \in S \wedge x \in R\}$, is the set of integers. Choices b, c, and d are incorrect because neither set contains complex or irrational numbers.

62. The correct answer is D. Consider $x = -1. (-1)^3 = -1 \ngeq 0$. All of the others are true.

63. The correct answer is B. $\forall x \exists y \sim L(x,y)$ can be translated to "For every x there exists a y such that x does not like y." In plain English that statement is equivalent to "Nobody likes everybody."

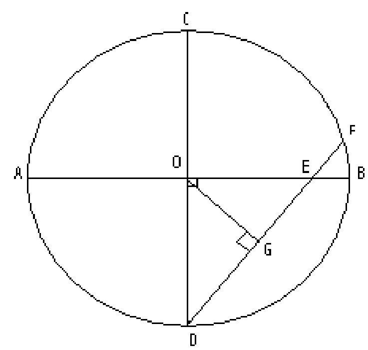
64. The correct answer is C. If $a|b$ and $a|c$ then $b = ak$ and $c = am$ for some $k, m \in \mathbb{Z}$. Thus $b + c = ak + am = a(k+m)$. Since $k + m \in \mathbb{Z}$, $a|(b+c)$. A is not true, if $a|b$ then $a|bc$ whether $a|c$ is true or not. B is not true, $a \equiv 0 (\text{mod } m)$ if $m|a$. D is not true. For example if $a = 6$, $b = 4$ and $c = 3$. $6 \mid 4 \cdot 3$ and $6 \nmid 4$, but $6 \nmid 3$.

65. The correct answer is D. To find $\int \frac{4x^3+2}{x^4+2x}dx$, let $u = x^4 + 2x$ so $du = 4x^3 + 2$. Therefore, $\int \frac{4x^3+2}{x^4+2x}dx = \int \frac{1}{u}du$ =ln|u|+$C = \ln|x^4 + 2x| + C$. This could also be determined by taking the derivative of all the answers to see which matches the integrand.

66. The correct answer is C. Construct the altitude of ΔDOF from vertex O to the opposite side $\overline{DF}$. Call the altitude $\overline{OG}$ (see illustration).

Since $\overline{OG}$ is an altitude to the hypotenuse of right triangle ΔDOF, the two smaller triangles formed are similar (and thus proportional) to ΔDOF. We have the proportion:

$$\frac{DE}{DO} = \frac{DO}{DG}$$



We also know from the geometry of a circle that if a segment containing the center O is perpendicular to a chord, then the chord is bisected. Thus G is the midpoint of DF, so we know DG = 4

Letting DO = r, DE = 6, DG = 4:

$$\Rightarrow \quad \frac{6}{r} = \frac{r}{4} \text{ or } r^2 = 24$$

Area is therefore 24π .

67. The correct answer is A. Of the 4 conic sections (parabola, circle, ellipse, hyperbola), the ellipse is the only one that can satisfy the given conditions. Upon inspection it is clear that the major axis is horizontal, with a vertex at (4,0), and the minor axis is vertical, with a vertex of (0,–2). The center must then be at (4,–2)

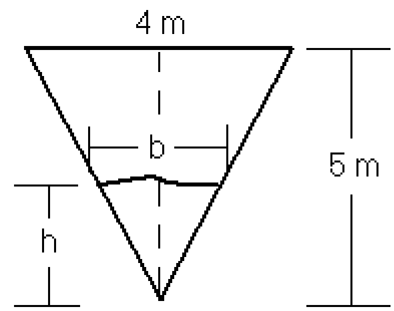
$$\frac{(x-4)^2}{4^2} + \frac{(y+2)^2}{2^2} = 1$$

Multiplying both sides by 16 and expanding the squares yields the answer A.

68. The correct answer is C. The only complex number listed that does not equal 1 when multiplied by itself 6 times is C.

69. The correct answer is D. The volume of water in the trough may be expressed as $V = \frac{1}{2}lbh$ where the length l is constant, but base b and height h depend on time. Using the product rule to differentiate the volume equation with respect to time yields: $\frac{dV}{dt} = \frac{1}{2}l\left(\frac{db}{dt}h + b\frac{dh}{dt}\right)$

To find a relation between the base and height, resort to the proportionality of similar triangles. From the diagram below, we can see that $\frac{4}{5} = \frac{b}{h}$ and thus $b = \frac{4}{5}h.$

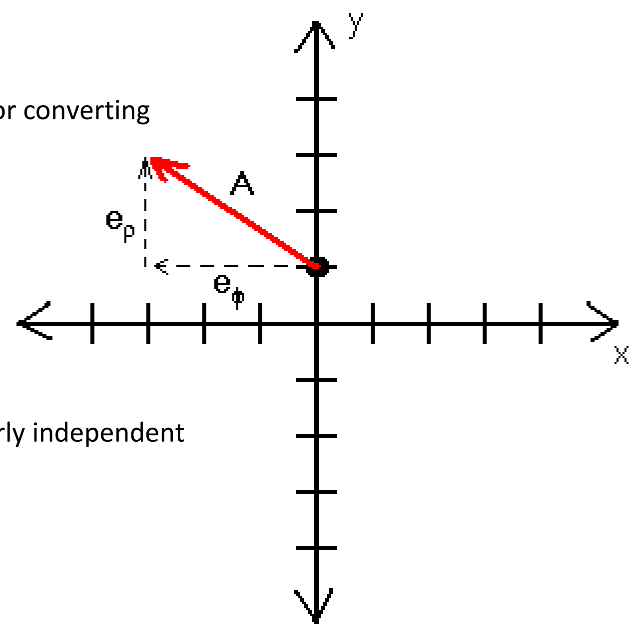


Differentiating with respect to time gives $\frac{db}{dt} = \frac{4}{5}\frac{dh}{dt}$.
Substituting into the volume rate equation yields $\frac{dV}{dt} = \frac{1}{2}l\left(\frac{4}{5}\frac{dh}{dt}h + \frac{4}{5}h\frac{dh}{dt}\right)$

Now letting $\frac{dV}{dt} = 5$, $h = 3$, and $l = 10$ and solving for $\frac{dh}{dt}$ yields $\frac{5}{24}$..

70. The correct answer is A. The first iteration has a shaded area of $\frac{1}{4}$.
The second iteration will add 3 triangles of area $\left(\frac{1}{4}\right)^2$.
The third iteration will add 9 triangles of area $\left(\frac{1}{4}\right)^3$.
As the pattern continues, the kth iteration will add 3^{k-1} triangles of area $\left(\frac{1}{4}\right)^k$ each, which we may express as $3^{k-1}\left(\frac{1}{4}\right)^k$ or equivalently $\frac{1}{4}\left(\frac{3}{4}\right)^{k-1}$. The total area for the nth iteration is then a summation over all previous terms: $\sum_{k=1}^{n}\frac{1}{4}\left(\frac{3}{4}\right)^{k-1}$.

71. The correct answer is A. The z component is unchanged. For converting the ρ and ϕ components, it helps to draw the projection in the x-y plane (seen to the right)



At the point given, the ρ component is along the y-direction, so the y component is 2.
The ϕ component falls along the negative x direction, so the x component is -3.

72. The correct answer is D. Recall that a basis must be a linearly independent set that spans the vector space (meaning that every vector may be represented by a linear combination of basis vectors).
Answer A is a basis for $\mathbb{R}^3$
Answer B is a basis for all degree 4 polynomials
Answer C is a basis, and has application in Fourier Series
Answer D is not a basis. The vectors are not linearly independent since they are scalar multiples of each other.